Ü b u n g s a r b e i t

Aufgabe 1

Bestimmen Sie die Ableitungen der folgenden Funktionen mit Hilfe der Faktorregel und der Summenregel.

a) $f(x) = 1 - 5x^3$ **b)** $f(x) = \frac{4}{x} + x$ **c)** $f(x) = \frac{1}{2x} + \left(\frac{x}{2}\right)^2$

d) $f(x) = \frac{10}{3}\sqrt{x}$ **e)** $f(x) = 4\sqrt{x} - \frac{5}{\sqrt{x}}$ **f)** $f(x) = ax^3 + bx^2 + cx + d$

g) $f(x) = \sin x - \cos x$ **h)** $f(x) = 2\cos x - 4x^2 + 2\sqrt[3]{x^2} - \frac{\sin x}{2}$

i) $f(x) = \frac{\cos x}{4} - \frac{1}{x} - \left(\sqrt{x} + 2\right)^2 + 16x^5$

j) $f(x) = 8x^3 + \frac{5}{x^2} - \frac{7}{x^4} + \frac{3}{\sqrt[6]{x^5}} + \frac{\sqrt[3]{x^5} \cdot \sqrt[4]{x^6}}{\sqrt{x^3}}$

Aufgabe 2

Bestimmen Sie die Ableitungen der folgenden Funktionen mit Hilfe der Produktregel.

a) $f(x) = x^2(1 - 3x^2)$ **b)** $f(x) = 3x^2\sqrt{x}$ **c)** $f(x) = (1 - 2x + x^2)\sin x$

d) $f(x) = \sin x \cdot \cos x$ **e)** $f(x) = (x^5 - x^2)(x^2 - x^4)$ **f)** $f(x) = (\cos x)^3$

g) $f(x) = 4\sqrt{x}\,\sin x \cos x$ **h)** $f(x) = 3x^4\cos x - 4x^5\sin x$

Aufgabe 3

Bestimmen Sie die Ableitungen der folgenden Funktionen mit Hilfe der Quotientenregel.

a) $f(x) = \frac{x^2 - x - 6}{x^2 + x - 6}$ **b)** $f(x) = \frac{2\sqrt{x}}{1 - \sqrt{x}}$ **c)** $f(x) = \tan x$ **d)** $f(x) = \frac{(x-1)^2}{\sin x}$

e) $f(x) = \frac{3}{x^2 + 1}$ **f)** $f(x) = \frac{x^3\sqrt{x}}{1 + x^2}$ **h)** $f(x) = \frac{(\cos x) + 1}{(\cos x) - 1}$ **h)** $f(x) = \frac{4x^3 - 5\sin x}{4x^3 + 5\cos x}$

Aufgabe 4

Bestimmen Sie die Ableitungen der folgenden Funktionen mit Hilfe der Kettenregel und eventuell mit Hilfe der anderen Regeln.

a) $f(x) = (4x^5 - 3x^4 + 46)^{12}$

b) $f(x) = \sqrt{4x^3 - 5x^2}$

c) $f(x) = \dfrac{\sin 5x}{x^2 - 4}$

d) $f(x) = \sqrt[5]{(x^3 - 4x^2 + 18x)^4}$

e) $f(x) = [\sin(x^2 - 1)] \cdot \cos 8x$

f) $f(x) = \dfrac{(x^7 - 6x^4)^{15}}{\sin(x^3 - 4x^2)}$

Aufgabe 5

Bestimmen Sie die Ableitungen der folgenden Funktionen.

a) $f(x) = \dfrac{1}{7x^2} + \dfrac{x^4}{4} - \sin x \tan x$

b) $f(x) = \dfrac{4}{3x^5 + 9} \cdot \dfrac{\cos x}{x^3\sqrt{x} - x^2 + 5}$

c) $f(x) = \dfrac{(\sin x)^2 + (\tan x)^2}{(x^5 - 3x^2 + 4\sqrt{x})\cos x}$

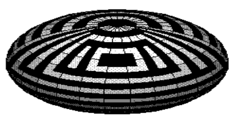
d) $f(x) = \dfrac{(4x^5 + 18x^4 - 7x^3 + 2x^2 - 3x + 7)^2}{3x^9 - 5x^4}$

e) $f(x) = \dfrac{\sin x \cdot \cos x \cdot \tan x}{(\cot x) \cdot 3x^{11}}$

f) $f(x) = \dfrac{\cot x}{3x^3}$

g) $f(x) = \dfrac{\cot x}{\sqrt{x + 1}}$

h) $f(x) = \dfrac{3x^5 \sin x}{\cos 3x^5}$



L ö s u n g e n

Aufgabe 1

a) $f(x) = 1 - 5x^3 \quad \Rightarrow \quad \underline{\underline{f'(x) = -15x^2}}$

b) $f(x) = \frac{4}{x} + x = 4x^{-1} + x \quad \Rightarrow \quad f'(x) = -4x^{-2} + 1 = \underline{\underline{1 - \frac{4}{x^2}}}$

c) $f(x) = \frac{1}{2x} + \left(\frac{x}{2}\right)^2 = \frac{1}{2}x^{-1} + \frac{1}{4}x^2 \quad \Rightarrow \quad f'(x) = -\frac{1}{2}x^{-2} + \frac{1}{2}x = \underline{\underline{\frac{1}{2}x - \frac{1}{2x^2}}}$

d) $f(x) = \frac{10}{3}\sqrt{x} = \frac{10}{3}x^{\frac{1}{2}} \quad \Rightarrow \quad f'(x) = \frac{5}{3}x^{-\frac{1}{2}} = \underline{\underline{\frac{5}{3\sqrt{x}}}}$

e) $f(x) = 4\sqrt{x} - \frac{5}{\sqrt{x}} = 4x^{\frac{1}{2}} - 5x^{-\frac{1}{2}} \quad \Rightarrow$

$f'(x) = 2x^{-\frac{1}{2}} + \frac{5}{2}x^{-\frac{3}{2}} = \frac{2}{\sqrt{x}} + \frac{5}{2\sqrt{x^3}} = \frac{2}{\sqrt{x}} + \frac{5}{2x\sqrt{x}} = \underline{\underline{\frac{4x+5}{2x\sqrt{x}}}}$

f) $f(x) = ax^3 + bx^2 + cx + d \quad \Rightarrow \quad \underline{\underline{f'(x) = 3ax^2 + 2bx + c}}$

g) $f(x) = \sin x - \cos x \quad \Rightarrow \quad \underline{\underline{f'(x) = \cos x + \sin x}}$

h) $f(x) = 2\cos x - 4x^2 + 2\sqrt[3]{x^2} - \frac{\sin x}{2} = 2\cos x - 4x^2 + 2x^{\frac{2}{3}} - \frac{1}{2}\sin x \quad \Rightarrow$

$f'(x) = -2\sin x - 8x + 1\frac{1}{3}x^{-\frac{1}{3}} - \frac{1}{2}\cos x = \underline{\underline{-2\sin x - 8x + \frac{4}{3\sqrt[3]{x}} - \frac{1}{2}\cos x}}$

i) $f(x) = \frac{\cos x}{4} - \frac{1}{x} - (\sqrt{x} + 2)^2 + 16x^5 = \frac{1}{4}\cos x - x^{-1} - (x + 4\sqrt{x} + 4) + 16x^5$

$= \frac{1}{4}\cos x - x^{-1} - x - 4x^{\frac{1}{2}} - 4 + 16x^5 \quad \Rightarrow$

$f'(x) = -\frac{1}{4}\sin x + x^{-2} - 1 - 2x^{-\frac{1}{2}} + 80x^4 = \underline{\underline{-\frac{1}{4}\sin x + \frac{1}{x^2} - \frac{2}{\sqrt{x}} + 80x^4 - 1}}$

j) $f(x) = 8x^3 + \frac{5}{x^2} - \frac{7}{x^4} + \frac{3}{\sqrt[6]{x^5}} + \frac{\sqrt[3]{x^5} \cdot \sqrt[4]{x^6}}{\sqrt{x^3}}$

$= 8x^3 + 5x^{-2} - 7x^{-4} + 3x^{-\frac{5}{6}} + x^{\frac{5}{3}} \cdot x^{\frac{3}{2}} \cdot x^{-\frac{3}{2}}$

$= 8x^3 + 5x^{-2} - 7x^{-4} + 3x^{-\frac{5}{6}} + x^{\frac{5}{3}}$

$f^{\mu}(x) = 24x^2 - 10x^{-3} + 28x^{-5} - \frac{15}{6}x^{-\frac{11}{6}} + \frac{5}{3}x^{\frac{2}{3}}$

$= \underline{\underline{24x^2 - \frac{10}{x^3} + \frac{28}{x^2} - \frac{15}{6\sqrt[6]{x^{11}}} + \frac{5}{3}\sqrt[3]{x^2}}}$

Aufgabe 2

a) $f(x) = x^2(1-3x^2)$

$u(x) = x^2 \quad u^{\mu}(x) = 2x \quad v(x) = 1 - 3x^2 \quad v^{\mu}(x) = -6x$

$f^{\mu}(x) = u(x) \cdot v^{\mu}(x) + u^{\mu}(x) \cdot v(x)$

$= x^2(-6x) + 2x(1 - 3x^2) = -6x^3 - 2x - 6x^3 = \underline{\underline{-12x^3 + 2x}}$

b) $f(x) = 3x^2\sqrt{x}$

$u(x) = 3x^2 \quad u^{\mu}(x) = 6x \quad v(x) = \sqrt{x} \quad v^{\mu}(x) = \frac{1}{2\sqrt{x}}$

$f^{\mu}(x) = u(x) \cdot v^{\mu}(x) + u^{\mu}(x) \cdot v(x)$

$= 3x^2 \cdot \frac{1}{2\sqrt{x}} + 6x\sqrt{x} = \frac{3}{2}x^{\frac{3}{2}} + 6x^{\frac{3}{2}} = \frac{15}{2}x^{\frac{3}{2}} = 7\frac{1}{2}\sqrt{x^3} = \underline{\underline{7\frac{1}{2}x\sqrt{x}}}$

c) $f(x) = (1 - 2x + x^2)\sin x$

$u(x) = 1 - 2x + x^2 \quad u^{\mu}(x) = -2 + 2x \quad v(x) = \sin x \quad v^{\mu}(x) = \cos x$

$f^{\mu}(x) = u(x) \cdot v^{\mu}(x) + u^{\mu}(x) \cdot v(x)$

$= (1 - 2x + x^2)\cos x + (-2 + 2x)\sin x = \underline{\underline{(x^2 - 2x + 1)\cos x + 2(x-1)\sin x}}$

d) $f(x) = \sin x \cdot \cos x$

$u(x) = \sin x \qquad u'(x) = \cos x \qquad v(x) = \cos x \qquad v'(x) = -\sin x$

$$f'(x) = u(x) \cdot v'(x) + u'(x) \cdot v(x)$$
$$= \sin x\,(-\sin x) + \cos x \cos x = \underline{\underline{\cos^2 x - \sin^2 x}}$$

e) $f(x) = (x^5 - x^2)(x^2 - x^4)$

$u(x) = x^5 - x^2 \qquad u'(x) = 5x^4 - 2x \qquad v(x) = x^2 - x^4 \qquad v'(x) = 2x - 4x^3$

$$f'(x) = u(x) \cdot v'(x) + u'(x) \cdot v(x)$$
$$\underline{\underline{= (x^5 - x^2)(2x - 4x^3) + (5x^4 - 2x)(x^2 - x^4)}}$$

f) $f(x) = (\cos x)^3 = (\cos x)^2 \cos x = f_1(x)\, f_2(x)$

mit $f_1(x) = (\cos x)^2$ und $f_2(x) = \cos x$

$u_1(x) = \cos x \qquad u_1'(x) = -\sin x \qquad v_1(x) = \cos x \qquad v_1'(x) = -\sin x$

$$f_1'(x) = u_1(x)\, v_1'(x) + u_1'(x)\, v_1(x)$$
$$= \cos x\,(-\sin x) + (-\sin x \cos x) = -2 \sin x \cos x$$

$f(x) = (\cos x)^2 \cos x = u(x)\, v(x)$

$u(x) = (\cos x)^2 \qquad u'(x) = -2 \sin x \cos x \qquad v(x) = \cos x \qquad v'(x) = -\sin x$

$$f'(x) = u(x) \cdot v'(x) + u'(x) \cdot v(x)$$
$$= (\cos x)^2 (-\sin x) + (-2 \sin x \cos x) \cos x = -\sin x \cos^2 x - 2 \sin x \cos^2 x$$
$$\underline{\underline{= -3 \sin x \cos^2 x}}$$

g) $f(x) = 4\sqrt{x}\, \sin x \cos x$

$u(x) = 4\sqrt{x} \qquad u'(x) = \dfrac{2}{\sqrt{x}} \qquad v(x) = \sin x \cos x \qquad v'(x) = \cos^2 x - \sin^2 x$ (S. Aufgabe 2d)

$$f'(x) = u(x)\, v'(x) + u'(x)\, v(x)$$
$$\underline{\underline{= 4\sqrt{x}\,(\cos^2 x - \sin^2 x) + \frac{2}{\sqrt{x}} \sin x \cos x}}$$

h) $f(x) = 3x^4 \cos x - 4x^5 \sin x = f_1(x) - f_2(x)$

mit $f_1(x) = 3x^4 \cos x$ und $f_2(x) = 4x^5 \sin x$

$u_1(x) = 3x^4 \quad u_1'(x) = 12x^3 \quad v_1(x) = \cos x \quad v_1'(x) = -\sin x$

$$f_1'(x) = u_1(x)\, v_1'(x) + u_1'(x)\, v_1(x)$$

$$= 3x^4(-\sin x) + 12x^3 \cos x = -3x^4 \sin x + 12x^3 \cos x$$

$u_2(x) = 4x^5 \quad u_2'(x) = 20x^4 \quad v_2(x) = \sin x \quad v_2'(x) = \cos x$

$$f_2'(x) = u_2(x)\, v_2'(x) + u_2'(x)\, v_2(x)$$

$$= 4x^5 \cos x + 20x^4 \sin x$$

$$f'(x) = f_1'(x) - f_2'(x)$$

$$= 3x^4 \sin x + 12x^3 \cos x - 4x^5 \cos x - 20x^4 \sin x$$

$$= -23x^4 \sin x + (12x^3 - 4x^5) \cos x = \underline{\underline{-23x^4 \sin x + 4(3x^3 - x^5) \cos x}}$$

Aufgabe 3

a) $f(x) = \dfrac{x^2 - x - 6}{x^2 + x - 6}$

$u(x) = x^2 - x - 6 \quad u'(x) = 2x - 1 \quad v(x) = x^2 + x - 6 \quad v'(x) = 2x + 1$

$$f'(x) = \frac{u'(x)\, v(x) - u(x)\, v'(x)}{[v(x)]^2}$$

$$= \frac{(2x - 1)(x^2 + x - 6) - (x^2 - x - 6)(2x + 1)}{(x^2 + x - 6)^2}$$

$$= \frac{2x^3 - x^2 + 2x^2 - x - 12x + 6 - 2x^3 + 2x^2 + 12x - x^2 + x + 6}{(x^2 + x - 6)^2}$$

$$= \underline{\underline{\frac{2x^2 + 12}{(x^2 + x - 6)^2}}}$$

b) $f(x) = \dfrac{2\sqrt{x}}{1 - \sqrt{x}}$

$u(x) = 2\sqrt{x} \quad u'(x) = \dfrac{1}{\sqrt{x}} \quad v(x) = 1 - \sqrt{x} \quad v'(x) = -\dfrac{1}{2\sqrt{x}}$

Fortsetzung 3 b)

$$f^{\mu}(x) = \frac{u^{\mu}(x)\, v(x) - u(x)\, v^{\mu}(x)}{[v(x)]^2} = \frac{\frac{1}{\sqrt{x}}(1 - \sqrt{x}) - [2\sqrt{x} \cdot (-\frac{1}{2\sqrt{x}})]}{(1 - \sqrt{x})^2}$$

$$= \frac{\frac{1}{\sqrt{x}} - 1 + 1}{(1 - \sqrt{x})^2} = \underline{\underline{\frac{1}{\sqrt{x}\,(1 - \sqrt{x})^2}}}$$

c) $f(x) = \tan x = \frac{\sin x}{\cos x}$

$u(x) = \sin x \qquad u^{\mu}(x) = \cos x \qquad v(x) = \cos x \qquad v^{\mu}(x) = -\sin x$

$$f^{\mu}(x) = \frac{u^{\mu}(x)\, v(x) - u(x)\, v^{\mu}(x)}{[v(x)]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \frac{\sin^2 x}{\cos^2 x} = \underline{\underline{1 + \tan^2 x = \frac{1}{\cos^2 x}}}$$

d) $f(x) = \frac{(x - 1)^2}{\sin x} = \frac{x^2 - 2x + 1}{\sin x}$

$u(x) = x^2 - 2x + 1 \qquad u^{\mu}(x) = 2x - 2 \qquad v(x) = \sin x \qquad v^{\mu}(x) = \cos x$

$$f^{\mu}(x) = \frac{u^{\mu}(x)\, v(x) - u(x)\, v^{\mu}(x)}{[v(x)]^2} = \underline{\underline{\frac{(2x - 2)\sin x - (x^2 - 2x + 1)\cos x}{\sin^2 x}}}$$

e) $f(x) = \frac{3}{x^2 + 1}$

$u(x) = 3 \qquad u^{\mu}(x) = 0 \qquad v(x) = x^2 + 1 \qquad v^{\mu}(x) = 2x$

$$f^{\mu}(x) = \frac{u^{\mu}(x)\, v(x) - u(x)\, v^{\mu}(x)}{[v(x)]^2} = \underline{\underline{\frac{-6x}{(x^2 + 1)^2} = \frac{-6x}{x^4 + 2x^2 + 1}}}$$

f) $f(x) = \frac{x^3 \sqrt{x}}{1 + x^2} = \frac{x^{\frac{7}{2}}}{1 + x^2}$

$u(x) = x^3 \sqrt{x} \qquad u^{\mu}(x) = \frac{7}{2} x^{\frac{5}{2}} = \frac{7}{2}\sqrt{x^5} = \frac{7}{2} x^2 \sqrt{x}$

$v(x) = 1 + x^2 \qquad v^{\mu}(x) = 2x$

Fortsetzung 3 f)

$$f^{\mu}(x) = \frac{u^{\mu}(x)\,v(x) - u(x)\,v^{\mu}(x)}{[v(x)]^2} = \frac{\frac{7}{2}x^2\sqrt{x}\,(1+x^2) - x^3\sqrt{x}\cdot 2x}{(1+x^2)^2}$$

$$= \frac{\frac{7}{2}x^2\sqrt{x} + \frac{7}{2}x^4\sqrt{x} - 2x^4\sqrt{x}}{(1+x^2)^2} = \frac{\frac{7}{2}x^2\sqrt{x} + \frac{3}{2}x^4\sqrt{x}}{(1+x^2)^2}$$

$$= \frac{(7+3x^2)\,x^2\sqrt{x}}{2\,(1+x)^2}$$

g) $f(x) = \frac{(\cos x)+1}{(\cos x)-1} = \frac{1+\cos x}{-1+\cos x}$

$u(x) = 1+\cos x \qquad u^{\mu}(x) = -\sin x \qquad v(x) = -1+\cos x \qquad v^{\mu}(x) = -\sin x$

$$f^{\mu}(x) = \frac{u^{\mu}(x)\,v(x) - u(x)\,v^{\mu}(x)}{[v(x)]^2} = \frac{(-\sin x)(-1+\cos x) - (1+\cos x)(-\sin x)}{(-1+\cos x)^2}$$

$$= \frac{-\sin x\cos x + \sin x + \sin x\cos x + \sin x}{(-1+\cos x)^2} = \frac{2\sin x}{(-1+\cos x)^2}$$

h) $f(x) = \frac{4x^3 - 5\sin x}{4x^3 + 5\cos x}$

$u(x) = 4x^3 - 5\sin x \qquad u^{\mu}(x) = 12x^2 - 5\cos x$

$v(x) = 4x^3 + 5\cos x \qquad v^{\mu}(x) = 12x^2 - 5\sin x$

$$f^{\mu}(x) = \frac{u^{\mu}(x)\,v(x) - u(x)\,v^{\mu}(x)}{[v(x)]^2} =$$

$$= \frac{(12x^2 - 5\cos x)(4x^3 + 5\cos x) - (4x^3 - 5\sin x)(12x^2 - 5\sin x)}{(4x^3 + 5\cos x)^2}$$

$$= [48x^5 - 20x^3\cos x + 60x^2\cos x - 25\cos^2 x$$
$$- (48x^5 - 60x^2\sin x - 20x^3\sin x + 25\sin^2 x)] \,/\, (4x^3 + 5\cos x)^2$$

$$= 48x^5 - 20x^3\cos x + 60x^2\cos x - 25\cos^2 x$$
$$- 48x^5 + 60x^2\sin x + 20x^3\sin x - 25\sin^2 x] \,/\, (4x^3 + 5\cos x)^2$$

$$= \frac{20x^3(\sin x - \cos x) + 60x^2(\sin x + \cos x) - 25}{(4x^3 + 5x\cos x)^2}$$

Aufgabe 4

a) $f(x) = (4x^5 - 3x^4 + 46)^{12}$

$$f'(x) = 12\,(4x^5 - 3x^4 + 46)^{11}\,(20x^4 - 12x^3) = \underline{\underline{48\,(4x^5 - 3x^4 + 46)^{11}\,(5x^4 - 3x^3)}}$$

b) $f(x) = \sqrt{4x^3 - 5x^2} = (4x^3 - 5x^2)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\,(4x^3 - 5x^2)^{-\frac{1}{2}}\,(12x^2 - 10x) = \underline{\underline{\frac{6x^2 - 5x}{\sqrt{4x^3 - 5x^2}}}}$$

c) $f(x) = \dfrac{\sin 5x}{x^2 - 4}$

$u(x) = \sin 5x \qquad u'(x) = 5\cos 5x \qquad v(x) = x^2 - 4 \qquad v'(x) = 2x$

$$\underline{\underline{f'(x) = \frac{(5\cos 5x)(x^2 - 4) - 2x\sin 5x}{(x^2 - 4)^2}}}$$

d) $f(x) = \sqrt[5]{(x^3 - 4x^2 + 18x)^4} = (x^3 - 4x^2 + 18x)^{\frac{4}{5}}$

$$f'(x) = \frac{4}{5}\,(x^3 - 4x^2 + 18x)^{-\frac{1}{5}}\,(3x^2 - 8x + 18) = \underline{\underline{\frac{12x^2 - 32x + 72}{5\sqrt[5]{x^3 - 4x^2 + 18x}}}}$$

e) $f(x) = [\sin(x^2 - 1)] \cdot \cos 8x$

$u(x) = \sin(x^2 + 1) \qquad u'(x) = 2x\cos(x^2 + 1) \qquad v(x) = \cos 8x \qquad v'(x) = -8\sin 8x$

$$f'(x) = 2x\,[\cos(x^2 + 1)]\cos 8x + [\sin(x^2 + 1)\cdot(-8)]\sin 8x$$

$$\underline{\underline{= 2x\,[\cos(x^2 + 1)]\cos 8x - 8\,[\sin(x^2 + 1)]\sin 8x}}$$

f) $f(x) = \dfrac{(x^7 - 6x^4)^{15}}{\sin(x^3 - 4x^2)}$

$u(x) = (x^7 - 6x^4)^{15} \qquad u'(x) = 15\,(x^7 - 6x^4)\,(7x^6 - 24x^3)$

$v(x) = \sin(x^3 - 4x^2) \qquad v'(x) = (3x^2 - 8x)\cos(x^3 - 4x^2)$

Fortsetzung von Aufgabe 4 f)

$$f^{\mu}(x) = [\,15\,(x^7 - 6x^4)^{14}\,(7x^6 - 24x^3)\,\sin(x^3 - 4x^2)$$
$$- (x^7 - 6x^4)^{15}\,(3x^2 - 8x)\,\cos(x^3 - 4x^2)\,] : [\sin^2(x^3 - 4x^2)\,]$$

Aufgabe 5

a) $f(x) = \dfrac{1}{7x^2} + \dfrac{x^4}{4} - \sin x \tan x = \dfrac{1}{7}x^{-2} + \dfrac{1}{4}x^4 - \dfrac{\sin x \cdot \sin x}{\cos x} = f_1(x) - f_2(x)$

mit $f_1(x) = \dfrac{1}{7}x^{-2} + \dfrac{1}{4}x^4$ und $f_2(x) = \dfrac{\sin x \cdot \sin x}{\cos x}$

$f_1^{\mu}(x) = -\dfrac{2}{7}x^{-3} + x^3 = x^3 - \dfrac{2}{7x^3}$

$f_2(x) = \dfrac{f_a(x)}{f_b(x)} = \dfrac{\sin x \cdot \sin x}{\cos x}$ Mit $f_a(x) = \sin x \sin x$ und $f_b(x) = \cos x$

Mit der Produktregel folgt: $f_a^{\mu}(x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x$

Mit der Quotientenregel folgt: $f_2^{\mu}(x) = \dfrac{2 \sin x \cos^2 x - (\sin^3 x)}{\cos^2 x}$

$$= 2 \sin x + \frac{\sin^3 x}{\cos^2 x}$$

Für die Ableitung $f^{\mu}(x)$ erhält man:

$$f^{\mu}(x) = f_1^{\mu}(x) - f_2^{\mu}(x) = x^3 - \frac{2}{7x^3} - 2\sin x - \frac{\sin^3 x}{\cos^2 x}$$

$$= x^3 - \frac{2}{7x^3} - 2\sin x - \tan^2 x \sin x = \underline{\underline{x^3 - \frac{2}{7x^3} - (2 + \tan^2 x)\sin x}}$$

b) $f(x) = \dfrac{4}{3x^5 + 9} \cdot \dfrac{\cos x}{x^3\sqrt{x} - x^2 + 5} = \dfrac{4 \cos x}{(3x^5 + 9)(x^{\frac{7}{2}} - x^2 + 5)}$

$$= \frac{4 \cos x}{3x^{\frac{17}{2}} + 9x^{\frac{7}{2}} - 3x^7 - 9x^2 + 15x^5 + 45}$$

Fortsetzung von Aufgabe 5 b)

$$f'(x) = [-4 \sin x\,(3x^5 + 9)(x^3\sqrt{x} - x^2 + 5)$$
$$- (\frac{51}{2}x^{\frac{15}{2}} + \frac{63}{2}x^{\frac{5}{2}} - 21x^6 - 18x + 75x^4)\,4\cos x\,]$$
$$:[(3x^5 + 9)(x^3\sqrt{x} - x^2 + 5)]^2$$

$$f'(x) = [-4 \sin x\,(3x^5 + 9)(x^3\sqrt{x} - x^2 + 5)$$
$$- (25\frac{1}{2}x^7\sqrt{x} + 31\frac{1}{2}x^2\sqrt{x} - 21x^6 + 75x^4 - 18x)\,4\cos x\,]$$
$$:[(3x^5 + 9)(x^3\sqrt{x} - x^2 + 5)]^2$$

---

c) $$f(x) = \frac{(\sin x)^2 + (\tan x)^2}{(x^5 - 3x^2 + 4\sqrt{x})\cos x} = \frac{\sin^2 x + \sin^2 x}{(x^5 - 3x^2 + 4\sqrt{x})\cos^3 x}$$

$$= \frac{2\sin^2 x}{(x^5 - 3x^2 + 4\sqrt{x})\cos^3 x} = \frac{u(x)}{v(x)}$$

$$u'(x) = 2(\sin x \cos x + \sin x \cos x) = 4\sin x \cos x$$

$(\cos^3 x)' = -3\sin x \cos^2 x$ (Siehe Aufgabe 2 f)

$$v'(x) = (5x^4 - 6x + \frac{2}{\sqrt{x}})\cos^3 x + (x^5 - 3x^2 + 4\sqrt{x})(-3\sin x \cos^2 x)$$

$$f'(x) = \{\,4\sin x \cos^4 x\,(x^5 - 3x^2 + 4\sqrt{x})$$
$$-2\sin^2 x\,[\,(5x^4 - 6x + \frac{2}{\sqrt{x}})\cos^3 x + (x^5 - 3x^2 + 4\sqrt{x})(-3\sin x \cos^2 x)]\,\}$$
$$:[(x^5 - 3x^2 + 4\sqrt{x})\cos^3 x\,]^2$$

---

d) $$f(x) = \frac{(4x^5 + 18x^4 - 7x^3 + 2x^2 - 3x + 7)^2}{3x^9 - 5x^4} = \frac{u(x)}{v(x)}$$

$$u'(x) = 2(4x^5 + 18x^4 - 7x^3 + 2x^2 - 3x + 7)(20x^4 + 72x^3 - 21x^2 + 4x - 3)$$

Mit Hilfe der Quotientenregel erhält man:

$$f'(x) = \{\,[\,(8x^5 + 36x^4 - 14x^3 + 4x^2 - 6x + 14)(20x^4 + 72x^3 - 21x^2 + 4x - 3)$$
$$\cdot (3x^9 - 5x^4)\,] - (4x^5 + 18x^4 - 7x^3 + 2x^2 - 3x + 7)^2 (27x^8 - 20x^3)\,]$$
$$:(3x^9 - 5x^4)^2$$

e) $f(x) = \dfrac{\sin x \cos x \tan x}{(\cot x) \cdot 3x^{11}} = \dfrac{\sin x \cos x \sin x \sin x}{3x^{11} \cos x \cos x} = \dfrac{\sin^3 x}{3x^{11} \cos x} = \dfrac{u(x)}{v(x)}$

$u(x) = \sin^2 x \sin x = u_1(x)\, u_2(x)$

$u_1^{\mu}(x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x \qquad u_2^{\mu}(x) = \cos x$

$u^{\mu}(x) = 2 \sin x \cos x \sin x + \sin^2 x \cos x = 3 \sin^2 x \cos x$

$v^{\mu}(x) = 3x^{11}(-\sin x) + 33x^{10} \cos x$

Mit Hilfe der Quotientenregel erhält man:

$$f^{\mu}(x) = \frac{3x^{11} 3 \sin^2 x \cos x \cos x - [\sin^3 x \cdot (-3x^{11} \sin x + 33x^{10} \cos x)]}{9x^{22} \cos^2 x}$$

$$= \frac{(9x^{11} \sin^2 x \cos^2 x + 3x^{11} \sin^4 x - 33x^{10} \sin^3 x \cos x}{9x^{22} \cos^2 x}$$

f) $f(x) = \dfrac{\cot x}{3x^3} = \dfrac{\cos x}{3x^3 \sin x} = \dfrac{u(x)}{v(x)}$

$v^{\mu}(x) = 9x^2 \sin x + 3x^3 \cos x$

$$f^{\mu}(x) = \frac{-\sin x (3x^3 \sin x) - (\cos x)(9x^2 \sin x + 3x^3 \cos x)}{9x^6 \sin^2 x}$$

$$= -\frac{3x^2 \sin^2 x + 9x^2 \sin x \cos x - 3x^3 \cos^2 x}{9x^6 \sin^2 x}$$

g) $f(x) = \dfrac{(\cot x)^2}{\sqrt{x+1}} = \dfrac{\cos^2 x}{\sqrt{x+1}\, \sin^2 x} = \dfrac{u(x)}{v(x)\, w(x)} = \dfrac{u(x)}{z(x)}$

mit $u(x) = \cos^2 x$, $v(x) = \sqrt{x+1}$, $w(x) = \sin^2 x$ und $z(x) = v(x)\, w(x)$

$u^{\mu}(x) = -2 \sin x \cos x \qquad v^{\mu}(x) = \dfrac{1}{2}(x+1)^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x+1}}$

$w^{\mu}(x) = 2 \sin x \cos x \qquad z^{\mu}(x) = \dfrac{\sin^2 x}{2\sqrt{x+1}} + \sqrt{x+1} \cdot 2 \sin x \cos x$

Fortsetzung Aufgabe 5 g)

$$f^{\mu}(x) = \frac{-(2\sin x\cos x)\sqrt{x+1}\,\sin^2 x - \left(\cos^2 x\right)\left(\frac{\sin^2 x}{2\sqrt{x+1}} + 2\sqrt{x+1}\,\sin x\cos x\right)}{\left(\sqrt{x+1}\,\sin^2 x\right)^2}$$

$$f^{\mu}(x) = \frac{-4\,(x+1)\,\sin^3 x\cos x - \cos^2 x\,[\sin^2 x + 4(x+1)\sin x\cos x)]}{2\sqrt{x+1}\,(x+1)\sin^4 x}$$

h) $f(x) = \frac{3x^5 \sin x}{\cos 3x^5} = \frac{u(x)}{v(x)}$

$u(x) = 3x^5 \sin x \qquad u^{\mu}(x) = 15x^4 \sin x + 3x^5 \cos x$

$v(x) = \cos 3x^5 \qquad v^{\mu}(x) = -15x^4 \sin 3x^5$

$$f^{\mu}(x) = \frac{(15x^4 \sin x + 3x^5 \cos x)\cos 3x^5 + 3x^5 \sin x\,(15x^4 \sin 3x^5)}{(\cos 3x^5)^2}$$